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P. Lamarque

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PRESENTATION OF A NEW GRAPH TO DETERMINE THE CONFIDENCE LIMITS FOR AN ESTIMATION OF THE MOST PROBABLE POPULATION USING THE PETERSEN METHOD

P. Lamarque

Centre for Hydrobiological Research
I.N.R.A., P.O. Box 3, Saint-Pée sur Nivelle - 643 ASCAIN

Abstract

The graph presented in this paper is rapid and easy to use. It provides symmetrical confidence limits for the most probable fish population as well as the level of precision of the census, thereby making it possible in the field to determine the advisability of extending fishing operations.

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"Presentation of a New Graph to Determine the Confidence Limits for an Estimation of the Most Probable Population Using the Petersen Method"

page 199, paragraph 2*, the equation should read:

$$P' = \frac{m}{e - 2\sigma e} \text{ and } P'' = \frac{m}{e + 2\sigma e}$$

$$\text{where } \sigma e = \sqrt{\frac{e(1-e)\,(P-n)}{n(P-1)}}$$

* See page 4 of translation --Tr.

1. INTRODUCTION

When the Petersen method is used for estimating a fish population, the latter must be broken down by species and length category in order to calculate its size and confidence limits, since fishing efficiency varies with these factors (Cuinat et al., 1075 a and b).

This requires a considerable number of calculations demanding a great deal of time even when a computer program or office calculators are pressed into service. For this reason, we prefer to use graphs instead of calculating standard deviations.

Since, however, the graphs currently employed have certain drawbacks, we set out to improve and simplify them.

2. METHODS CURRENTLY USED TO DETERMINE THE CONFIDENCE LIMITS WITH A 95 % RELIABILITY LEVEL

2.1. Standard Deviation of a Ratio (Timmermans, 1957)

The following symbols were adopted:

P = the most probable population

P' = the most probable population with indication of the upper limit

P" = the most probable population with indication of the lower limit

m = the number of fish marked and released after the first fishing operation

u = the number of unmarked fish caught during the second fishing operation

r = number of marked fish caught during the second fishing operation

n = total number of fish caught during the second fishing operation ($n = u + r$)

e = efficiency of the first fishing operation.

By taking into account the standard deviation of proportion e , we obtain:

$$P' = \frac{m}{e - 2\sqrt{e}} \quad \text{and} \quad P'' = \frac{m}{e + 2\sqrt{e}} \qquad \pm 2\sigma e$$

$$\text{where} \quad \sqrt{e} = \sigma e = \sqrt{\frac{e(1-e)(P-n)}{n(P-1)}}$$

Admittedly, the use of this method is long and tedious when the population lacks uniformity and has to be subdivided.

2.2. The Clopper and Pearson Graphs (1934, after Cuinat 1975, a and b) and **Cuinat Graph** (1975, a and b)

The Clopper and Pearson graphs (Fig. 1) were designed for large populations. The confidence limits are too far apart for relatively small populations.

Cuinat (Fig. 2) consequently modified them for small trout streams when yields from the first and second catches are esentially the same ($m \simeq n$).

These graphs have three drawbacks, however:

- They are not very precise for fish populations in excess of 200 individuals because of their graphic configuration;

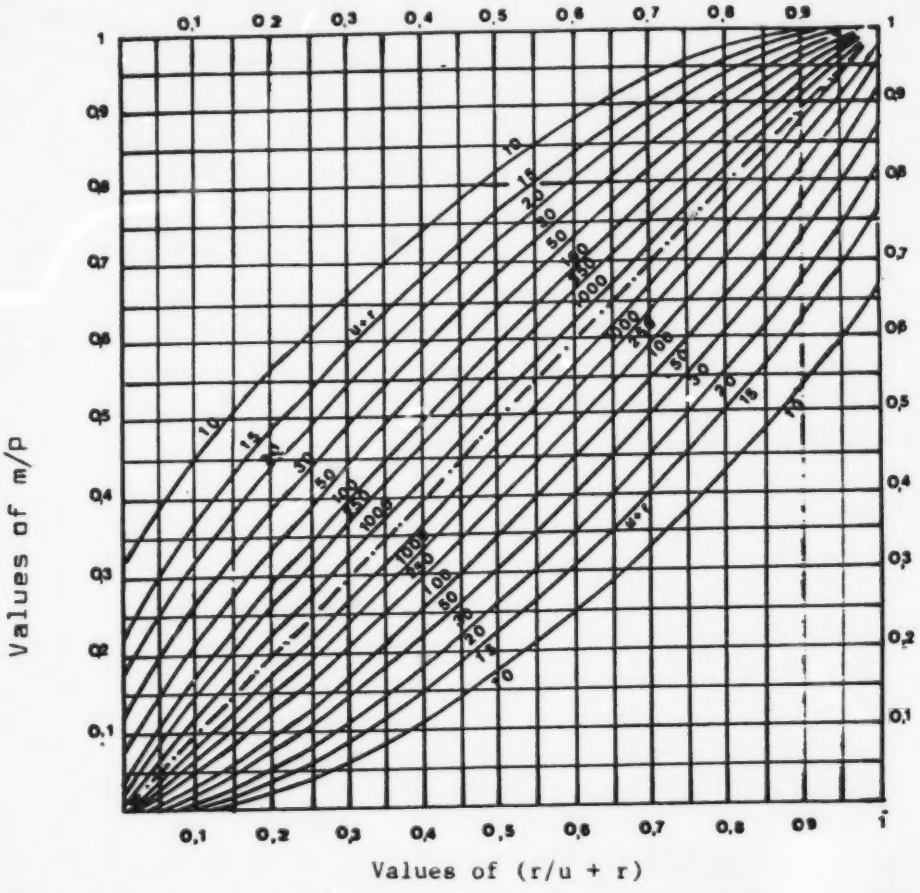

Fig. 1. Clopper and Pearson Graph (1934, after Cuinat, 1975 a and b)

Symbols Used:

m = number of fish of a given species and length category, marked and released after the first catch

r = number of marked fish of the same species and length category caught during the second fishing operation

u = number of unmarked fish of the same species and length category caught during the second fishing operation

P = estimate of the most probable number of these fish calculated by the Petersen method:

$$P = \frac{m}{r/(u + r)} = \frac{m\,(u + r)}{r}$$

Use of the Graph. Example:

Let m = 120, r = 60 and u = 40. r/u + r = 0.6 and P (the most probable number) = m/0.6 = 200. The two curves representing (u + r) = 100 intersect the vertical line drawn from point 0.6 on the X axis at values of 0.49 and 0.69, respectively, on the Y axis. The confidence limits about P = 200 are therefore P' = 120/0.49 = 245 and P" = 120/0.69 = 174.

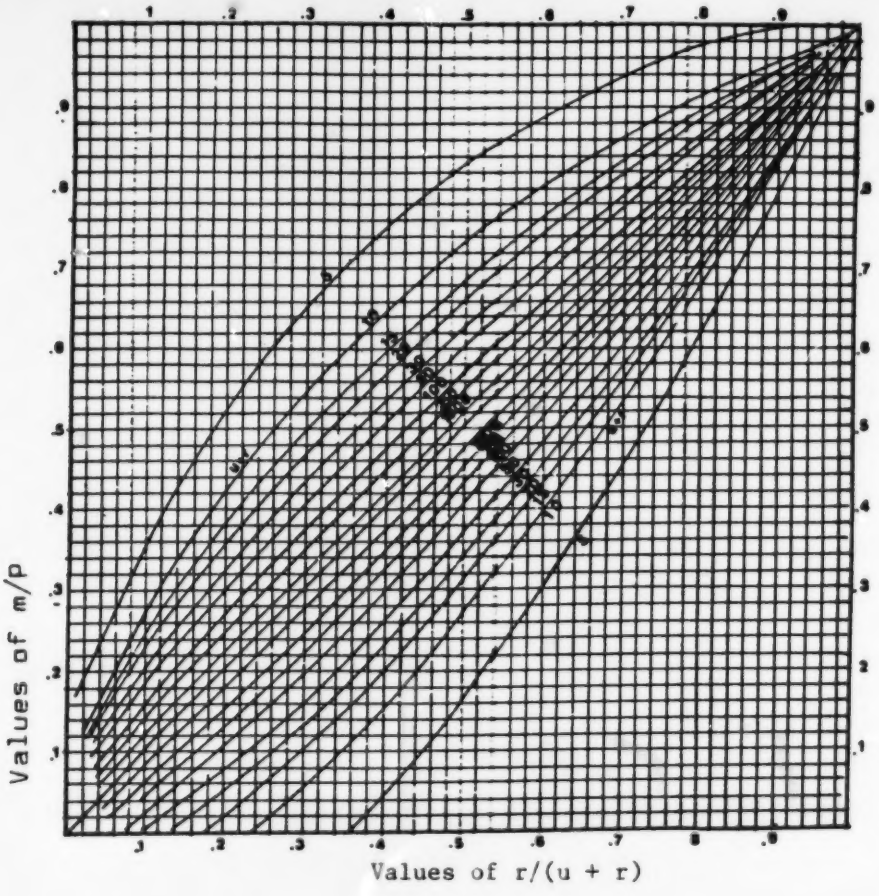

Fig. 2. Cuinat Graph (1975 a and b)

Same symbols and explanations for use as in Fig. 1.

- Like standard deviation calculations, they do not provide symmetrical confidence limits for the most probable population;
- They are more difficult to read than the graph we propose below.

3. ESTABLISHING THE LAMARQUE GRAPH

We shall keep the symbols given above. In order to reduce the bias created by small samples, Chapman (1951) introduced a corrective to Petersen's equation which, in fact, alters the results only minimally.

Instead of $P = \frac{mn}{r}$, he suggested: $P = \frac{(m + 1)(n + 1)}{(r + 1)}$.

The population variance may be calculated from this ratio using the following equation (Vincent, 1971)

P var. $= \frac{P^2u}{(n + 1)(r + 2)}$. Thus, for 2 standard deviations:

$$2\sqrt{\frac{P^2u}{(n + 1)(r + 2)}}.$$

The populations with indication of their confidence limits are:

$$P' = P + 2\sqrt{\frac{P^2u}{(n + 1)(r + 2)}}$$

$$\text{and } P'' = P - 2\sqrt{\frac{P^2u}{(n + 1)(r + 2)}}.$$

This yields symmetrical limits for the estimated population.

If now the two standard deviations are considered a proportion ρ of the estimated population (accuracy of the measurement), we can write:

$$2\sqrt{\frac{P^2(n-r)}{(n+1)(r+2)}} = \rho P \qquad \text{(u is equal to n-r).}$$

By developing and simplifying, we end up with:

$$\frac{4(n-r)}{(n+1)(r+2)} = \rho^2 \text{ , a ratio that is independent of P.}$$

When the ratio $\frac{r}{n}$ (efficiency of the first catch) is introduced, a function of the following type is obtained:

$n = f(\frac{r}{n})$.

To produce the graph, the different values of ρ and, for each ρ value, the different values of n, are programmed.

This yields the r values from which the ratio $\frac{r}{n}$ may be calculated.

All that remains to be done after that is to draw the various ρ curves, placing the $\frac{r}{n}$ values on the x axis and the corresponding n values on the y axis (Fig. 3).

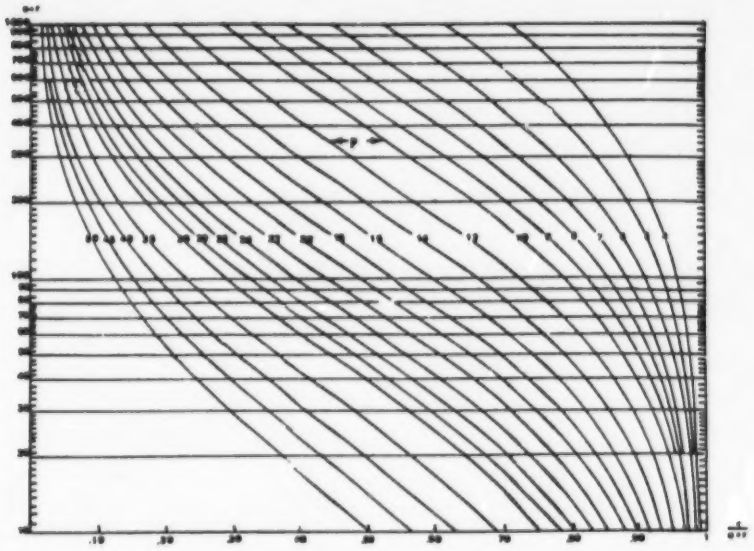

Fig. 3. Lamarque's Graph

m = the number of fish of a given species and length category, marked and released after the first catch

r = the number of marked fish of a given species and length category caught during the second fishing operation

u = number of unmarked fish of a given species and length category caught during the second fishing operation

P = estimate of the most probable number of these fish obtained by the Petersen method, as modified by Chapman:

$$P = \frac{(m + 1)(u + r + 1)}{(r + 1)}$$

p = accuracy of the measurement in percentage terms.

Use of the Graph. Example.

Find a point where a value on the ordinate scale (u + r) intersects a value on the abscissa scale $\frac{r}{u + r}$. The precision curve ρ on which the point lies gives the confidence limits for the population in percentage terms with a 95 % reliability level.

Let P = 200; u + r = 100; $\frac{r}{u + r} = 0.6$. The corresponding curve is 16, which means that the confidence limits are, respectively, $\pm$ 0.16 P: P' = P + 0.16 P = 232 and P" = P - 0.16 P = 168.

4. COMPARISON OF CONFIDENCE LIMITS OBTAINED BY DIFFERENT METHODS

The confidence limits obtained with the new graph are always closer together than those obtained with Clopper and Pearson's graph, yet, slightly wider apart than encountered with the other methods. For small populations, however, they are very similar to those yielded by the standard deviation of the proportion and by Cuinat's graph, as may be seen from the examples given in Table 1.

Table 1. Examples of Populations with Indication of Their Confidence Limits Established by Different Methods

	small populations	large populations
Census findings	P = 20; m = 12; r = 6 u = 4; (u + r) = 10 r/(u + r) = 0.6	P = 400; m = 240; r = 120 u = 80; (u + r) = 200 r/(u + r) = 0.6
Cuinat's Graph	P' = 30 P" = 15	P' = 436 P" = 369
Standard deviation of a proportion	P' = 32 P" = 14	P' = 435 P" = 369
Clopper and Pearson's Graph	P' = 48 P" = 13	P' = 470 P" = 363
Lamarque's new graph	P' = 29 P" = 11 precision: 0.42	P' = 446 P" = 354 precision: 0.11

Comment: With the exception of Clopper and Pearson's Graph for small populations, the differences between the various methods remain negligible considering the lack of precision of the census-taking method itself.

5. HOW TO READ LAMARGUE'S GRAPH

A graduated ruler is placed on a value on the ordinate scale $n = u + r$.

The value on the abscissa scale $\frac{r}{u + r}$ is read with the aid of the ruler.

The precision curve p containing the point where the two values intersect provides the confidence limits, e.g.,

$$P = 400;\ n = u + r = 200;\ \frac{r}{u + r} = 0.6.$$

The point where ordinate value 200 intersects abscissa value 0.6 lies on the 11.5 precision curve.

The upper and lower limits of the population, with a 95 % reliability level, are as follows:

$P' = 400 + (0.115 \times 400) = 446$

$P'' = 400 - (0.115 \times 400) = 354.$

6. VALUE OF THE NEW GRAPH

In addition to being a quick method and producing symmetrical limits, the graph also immediately provides the level of precision of the population estimate.

Robson and Reger (1964) have shown that a study may be more efficiently conducted if the level of precision is known.

They recommend either aiming for or being satisfied with the following levels of precision:

10 percent for scientific studies

25 percent for development work

50 percent for forecasting purposes

and suggest a method for determining the number of fish to be marked in order to obtain the selected level of precision from graphs using P and m.

With this new graph, the level of precision based on the first two catches can be determined in the field. If it is lower than needed for the objective set, then the second-catch fish may be marked before being released and a third catch may be made.

This paper focused exclusively on the application of the Petersen method to fish, but the graph discussed may naturally be used for other animal species.

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